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$(\alpha, \beta, \gamma)-$ Derivations of Diassociative Algebras

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ABSTRACT

In this research we introduce a generalized derivations of diassociative algebras and study its properties. This generalization depends on some parameters. In this paper we specify all possible values of the parameters. We also provide all the generalized derivations of low-dimensional complex diassociative algebras.

Keywords: Derivation, Diassiciative algebras, generalized derivations.

1. Introduction

There are several approaches to the study of generalized derivations on Lie algebras. For example, Hartwig et al. (2006) defined the generalized derivation of a Lie algebra L as a linear operator A satisfying the property $A[x, y] = [Ax, \tau y] + [\sigma x, Ay]$, where τ, σ are fixed elements of $End\ L$. Such derivations have been called (σ, τ)-derivations of L. Bresar (1991) considered the following generalization: a linear transformation A of a Lie algebra L is said to be a generalized derivation if there exists $B \in DerL$ such that for all $x, y \in L$, the condition $A[x, y] = [Ax, y] + [x, By]$ holds. Leger and Luks (2000) studied a more general version defining the generalized derivation as $A \in EndL$ such that there exist $B, C \in EndL$ possessing the property $C[x, y] = [Ax, y] + [x, By]$. Novotnŷ and Hrivnák (2008) introduced a new version of a generalization of Lie algebra derivations and use them in algebraic and geometric classification problems in low-dimensional cases. Our interest in this paper is to study the generalized derivations of finite dimensional diassociative algebras. The algebra of derivations and generalized derivations are very useful in algebraic and geometric classification problems of algebras.

Around 1990, Loday (1993) introduced the notions of Leibniz and diassociative algebras (associative dialgebras). The Leibniz algebra is a generalization of Lie algebra where the skew-symmetricity of the bracket is suppressed and the Jacobi identity is changed by the Leibniz identity. The notion of dialgebra is a generalization of associative algebra. Any associative algebra gives rise to Lie algebra as follows $[a, b] = ab - ba$. Diassociative algebra plays the role of associative algebra in this link for Leibniz algebra. Loday showed that if $(D, \dashv, \vdash)$ is a diassociative algebra structure on a vector space A with two associative products $\dashv$ and $\vdash$, then $[x, y] = x \dashv y - y \vdash x$ defines a Leibniz algebra structure on D and vice-versa the enveloping algebra of a Leibniz algebra has the structure of dialgebra (see Loday et al. (2001)). In this paper we deal with the problem of description of the $(\alpha, \beta, \gamma)-$ derivations of diassociative algebras. The concept of generalized derivation in this case can be easily imitated from the definition of the derivations of associative algebras. In the case $\alpha = \beta = \gamma = 1$, we get derivations of diassociative algebra, which have been studied in Abubakar et al. (2014). Here we give an algorithm to compute the derivations. We apply the algorithm in low-dimensional cases. The results of all applications are given in the form of tables. To obtain it we use the classification results of two and three-dimensional complex associative dialgebras from Rikhsiboev et al. (2010). and revised list of three-dimensional diassociative algebras from Rakhimov and Fiidow (2015).

2. Preliminaries

In this section, we shall start with simple definitions and facts needed later in the course of our discussions.

Definition 2.1. *Let D be a vector space over a field $\mathbb{K}$ equipped with two associative bilinear binary operations* $\dashv: D \times D \longrightarrow D$ *and* $\vdash: D \times D \longrightarrow D$

satisfying the axioms:

$$\begin{aligned} x \dashv (y \dashv z) &= x \dashv (y \vdash z), \\ (x \vdash y) \dashv z &= x \vdash (y \dashv z), \\ (x \vdash y) \vdash z &= (x \dashv y) \vdash z, \end{aligned}$$

for all $x, y, z \in D$. Then the triple $(D, \dashv, \vdash)$ is called a diassociative algebra.

The operations $\dashv$ and $\vdash$ are called left and right products, respectively.

Example 2.1. *Any associative algebra D is a diassociative algebra since we can put $x \dashv y = xy = x \vdash y$.*

Example 2.2. *Let $\mathbb{K}[x, y]$ is a polynomial algebras over a field $\mathbb{K}(char\mathbb{K} = 0)$ with two indeterminate x, y. We define the left product $\dashv$ and the right product $\vdash$ on $\mathbb{K}[x, y]$ as follows:*

$$f(x, y) \dashv g(x, y) = f(x, y)g(y, y)$$

and

$$f(x, y) \vdash g(x, y) = f(x, x)g(x, y).$$

Then $(\mathbb{K}[x, y], \dashv, \vdash)$ is a diassociative algebra (see Lin and Zhang (2010)).

Example 2.3. *A dialgebra D generates so called opposite dialgebra D^{op} over the same underlying $\mathbb{K}-$vector space and the left and the right product are given as:*

$$x \vdash^{'} y = y \dashv x \quad and \quad x \dashv^{'} y = y \vdash x.$$

Definition 2.2. *Let $(D_1, \dashv_1, \vdash_1)$, $(D_2, \dashv_2, \vdash_2)$ be a diassociative algebras over a field $\mathbb{K}$. Then a homomorphism from D_1 to D_2 is a $\mathbb{K}$- linear mapping $\phi : D_1 \to D_2$ such that*

$$\phi(x \dashv_1 y) = \phi(x) \dashv_2 \phi(y)$$

and

$$\phi(x \vdash_1 y) = \phi(x) \vdash_2 \phi(y)$$

for all $x, y \in D_1$.

Remark 2.1. *A bijective homomorphism is an isomorphism of D_1 and D_2.*

Definition 2.3. *A derivation of diassociative algebra D is a linear transformation $d : D \to D$ satisfying*

$$d(x \dashv y) = d(x) \dashv y + x \dashv d(y)$$

and

$$d(x \vdash y) = d(x) \vdash y + x \vdash d(y)$$

for all $x, y \in D$.

The set of all derivations of a diassociative algebra D is a subspace of $End_{\mathbb{K}}(D)$. This subspace equipped with the bracket $[d_1, d_2] = d_1 \circ d_2 - d_2 \circ d_1$ is a Lie algebra denoted by $DerD$. As it was mentioned early this case was studied in Abubakar et al. (2014).

3. Results

3.1 $(\alpha, \beta, \gamma)-$derivations of diassociative algebra

Several non-equivalent ways of generalizing Definition 2.3 of derivations have been given in Abubakar et al. (2014). We bring forward another type of generalization of the derivations.

A linear operator $d \in EndD$ is said to be (α, β, γ)-derivation of D, where α, β, γ are fixed elements of $\mathbb{K}$, if for all $x, y \in D$

$$\alpha d(x \dashv y) = \beta d(x) \dashv y + \gamma x \dashv d(y)$$

and

$$\alpha d(x \vdash y) = \beta d(x) \vdash y + \gamma x \vdash d(y).$$

The set of all $(\alpha, \beta, \gamma)-$derivations of D we denote by $Der_{(\alpha,\beta,\gamma)}D$. It is clear that $Der_{(\alpha,\beta,\gamma)}D$ is a linear subspace of $EndD$. The following proposition specifies all possible values of the triple (α, β, γ) for complex diassociative algebras.

Proposition 3.1. *Let D be a complex diassociative algebra. Then the values of α, β, γ in $Der_{(\alpha,\beta,\gamma)}D$ are distributed as follows:*

$$\begin{aligned}
Der_{(1,1,1)}D &= \big\{d \in EndD | d(x \star y) = d(x) \star y \\
&\qquad + x \star d(y)\big\}; \\
Der_{(1,1,0)}D &= \big\{d \in EndD | d(x \star y) = d(x) \star y\big\}; \\
Der_{(1,0,1)}D &= \big\{d \in EndD | d(x \star y) = x \star d(y)\big\}; \\
Der_{(1,0,0)}D &= \big\{d \in EndD | d(x \star y) = 0\big\}; \\
Der_{(0,1,1)}D &= \big\{d \in EndD | d(x) \star y = x \star d(y)\big\}; \\
Der_{(0,0,1)}D &= \big\{d \in EndD | x \star d(y) = 0\big\}; \\
Der_{(0,1,0)}D &= \big\{d \in EndD | d(x) \star y = 0\big\}; \\
Der_{(0,1,\delta)}D &= \big\{d \in EndD | d(x) \star y = \delta x \star d(y), \\
&\qquad \delta \in \mathbb{C} \setminus \{0, 1\}\big\};
\end{aligned}$$

where $\star = \dashv, \vdash$.

Proof. Let $\alpha \neq 0$. Applying the operator d to the identities of diassociative algebra we obtain the system of equations:

$$\beta(\beta-\alpha)=0 \text{ and } \gamma(\gamma-\alpha)=0.$$

Applying case by case consideration we obtain the following possible values of

$$(\alpha,\beta,\gamma):(\alpha,\alpha,\alpha),(\alpha,\alpha,0),(\alpha,0,\alpha) \text{ and } (\alpha,0,0).$$

Taking into account the fact that

$$Der_{(\alpha,\beta,\gamma)}D = Der_{(1,\beta/\alpha,\gamma)}D$$

we get the required first 4 equalities in Proposition 3.1.

Let now $\alpha = 0$. Then we have

$$(0,\beta,\beta),\ (0,\beta,0) \text{ if } \beta\neq 0$$

and

$$(0,0,\gamma) \text{ if } \beta = 0.$$

Hence, we get

$$Der_{(0,\beta,\gamma)}D = Der_{(0,1,\gamma/\beta)}D \text{ if } \beta\neq 0$$

and

$$Der_{(0,0,\gamma)}D = Der_{(0,0,1)}D \text{ if } \beta = 0 \text{ and } \gamma\neq 0.$$

Finally, if $\gamma = 0$ then $Der_{(0,0,0)}(D) = EndD$. □

Proposition 3.2. *Let D be diassociative algebra and d_1, d_2 be (α,β,γ)-derivation on D, then $[d_1,d_2] = d_1\circ d_2 - d_2\circ d_1$ is a $(\alpha^2,\beta^2,\gamma^2)$-derivation of D.*

Proof. Let us consider the following two equations

$$\alpha d_1(x\star y) = \beta d_1(x)\star y + \gamma x\star d_1(y) \quad (1)$$

and

$$\alpha d_2(x\star y) = \beta d_1(x)\star y + \gamma x\star d_2(y). \quad (2)$$

Case I: $\alpha\neq 0$.

$$\begin{aligned}
\alpha^2[d_1,d_2](x\star y) &= \alpha^2(d_1\circ d_2)(x\star y) - \alpha^2(d_2\circ d_1)(x\star y)\\
&= \alpha d_1(\alpha d_2(x\star y)) - \alpha d_2(\alpha d_1(x\star y))\\
&= \alpha d_1(\beta d_2(x)\star y + \gamma x\star d_2(y)) -\\
&\quad \alpha d_2(\beta d_1(x)\star y + \gamma x\star d_1(y))\\
&= \beta(\alpha d_1(d_2(x)\star y)) + \gamma(\alpha d_1(x d_2(y)))\\
&\quad -\beta(\alpha d_2(d_1(x)\star y) + \gamma\alpha d_2(x\star d_1(y))).
\end{aligned}$$

Finally we get

$$\alpha^2[d_1, d_2](x \star y) = \beta^2[d_1, d_2](x) \star y + \gamma^2 x \star [d_1, d_2](y). \tag{3}$$

Case II: $\alpha = 0$. Then from the equation (1) and (2) we get

$$\beta d_1(x) \star y = -\gamma x \star d_1 y \tag{4}$$

and

$$\beta d_2(x) \star y = -\gamma x \star d_2 y. \tag{5}$$

Therefore

$$\begin{aligned} \beta^2[d_1, d_2](x \star y) &= \beta^2(d_1 \circ d_2)(x) \star y - \beta^2(d_2 \circ d_1)(x) \star y \\ &= \beta^2(d_1 d_2)(x) \star y - \beta^2(d_2 d_1)(x) \star y \\ &= \beta(\beta d_1(d_2(x) \star y - \beta(\beta d_2(d_1(x) \star y. \end{aligned}$$

By using the equations from (4) and (5) we get

$$\begin{aligned} \beta^2[d_1, d_2](x) \star y &= \beta(-\gamma d_2)(x) d_1(y) + \beta(\gamma d_1)(x) d_2 y \\ &= -\gamma(\beta d_2(x) d_1(y) + \gamma(\beta d_1(x) d_2(y). \end{aligned}$$

Due to (4) and (5) again we obtain

$$\begin{aligned} \gamma^2 x \star d_2(d_1(y) - \gamma^2 x \star d_1(d_2(y)) &= \gamma^2 x[d_2(d_1(y) - d_1(d_2(y))] \\ &= \gamma^2 x \star [d_1, d_2](y). \end{aligned}$$

□

Remark 3.1. *For any $\alpha, \beta, \gamma \in \mathbb{C}$, the dimension of the vector space $Der_{(\alpha,\beta,\gamma)}D$ is an isomorphism invariant of diassociative algebras.*

3.2 (α, β, γ)-derivations of low-dimensional diassociative algebras

This section is devoted to the description of generalized derivations of two and three-dimensional complex diassociative algebras. Let $\{e_1, e_2, e_3, \cdots, e_n\}$ be a basis of an n-dimensional diassociative algebra D. Then

$$e_i \dashv e_j = \sum_{k=1}^{n} \gamma_{ij}^k e_k \quad \text{and} \quad e_s \vdash e_t = \sum_{l=1}^{n} \delta_{st}^l e_l,$$

where $i, j, s, t = 1, 2, ..., n$. The coefficients of the above linear combinations $\{\gamma_{ij}^k, \delta_{st}^l\} \in \mathbb{C}^{2n^3}$ are called the structure constants of D on the basis $\{e_1, e_2, e_3, \cdots, e_n\}$. A generalized derivation being a linear transformation of the vector space D is represented in a matrix form $[d_{ij}]_{i,j=1,2,...,n}$, i.e. $d(e_i) = \sum_{j=1}^n d_{ji}e_j$, $i = 1, 2, ..., n$. According to the definition of the generalized derivation, the entries d_{ij} $i, j = 1, 2, ..., n$, of the matrix $[d_{ij}]_{i,j=1,2,...,n}$ must satisfy the following systems of equations:

$$\sum_{t=1}^{n} (\alpha\gamma_{ij}^t d_{st} - \beta d_{ti}\gamma_{tj}^s - \gamma d_{tj}\gamma_{it}^s) = 0$$

for $i, j, s = 1, 2, 3, ..., n$

$$\sum_{l=1}^{n} (\alpha\delta_{st}^l d_{ml} - \beta d_{ls}\delta_{lt}^m - \gamma d_{lt}\delta_{sl}^m) = 0$$

for $s, t, m = 1, 2, 3, ..., n$.

It is observed that if the structure constants $\{\gamma_{ij}^k, \delta_{st}^l\}$ of a diassociative algebra D are given then in order to describe its generalized derivation one has to solve the system of equations above with respect to d_{ij} $i, j = 1, 2, \cdots, n$, which can be done by using a computer software. The values of structure constants γ_{ij}^k and δ_{st}^l for two and three-dimensional complex diassociative algebras we get from the classification results of Rikhsiboev et al. (2010) and Rakhimov and Fidow (2015) where $Dias_n^m$ denotes as m-th isomorphism class of n-dimensional complex diassociative algebras. The outputs of the study in dimension two and three are given by the following tables.

The (α, β, γ)-derivations of two-dimensional complex diassociative algebras are given as follows:

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}D$	Dim
$Dias_2^1$	(1,1,1)	$\begin{pmatrix}0 & 0\\ 0 & d_{22}\end{pmatrix}$	1
	(1,1,0)	$\begin{pmatrix}d_{11} & 0\\ 0 & d_{22}\end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix}d_{11} & 0\\ 0 & d_{11}\end{pmatrix}$	1
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	$\begin{pmatrix}0 & 0\\ d_{21} & d_{22}\end{pmatrix}$	2
	(0,1,0)	trivial	0
	(0,1,δ)	$\begin{pmatrix}-\delta d_{11} & 0\\ 0 & -\delta d_{11}\end{pmatrix}$	1
$Dias_2^2$	(1,1,1)	$\begin{pmatrix}0 & 0\\ 0 & d_{22}\end{pmatrix}$	1
	(1,1,0)	$\begin{pmatrix}d_{11} & 0\\ 0 & d_{11}\end{pmatrix}$	1
	(1,0,1)	$\begin{pmatrix}d_{11} & 0\\ 0 & d_{22}\end{pmatrix}$	2
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	trivial	0
	(0,1,0)	$\begin{pmatrix}0 & 0\\ d_{21} & d_{22}\end{pmatrix}$	2
	(0,1,δ)	$\begin{pmatrix}-\delta d_{11} & 0\\ 0 & -(1/\delta)d_{11}\end{pmatrix}$	1

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}D$	Dim
$Dias_2^3$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 \\ d_{21} & 2d_{11} \end{pmatrix}$	2
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 \\ d_{21} & d_{11} \end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 \\ d_{21} & d_{11} \end{pmatrix}$	2
	(1,0,0)	$\begin{pmatrix} d_{11} & 0 \\ d_{21} & 0 \end{pmatrix}$	2
	(0,1,1)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2
	(0,0,1)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2
	(0,1,0)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2
	(0,1,δ)	$\begin{pmatrix} -\delta d_{11} & 0 \\ d_{21} & d_{22} \end{pmatrix}$	3
$Dias_2^4$	(1,1,1)	$\begin{pmatrix} 0 & 0 \\ d_{21} & d_{22} \end{pmatrix}$	2
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{11} \end{pmatrix}$	1
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 \\ 0 & d_{11} \end{pmatrix}$	1
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	trivial	0
	(0,1,0)	trivial	0
	(0,1,δ)	$\begin{pmatrix} -\delta d_{11} & 0 \\ 0 & -\delta d_{11} \end{pmatrix}$	1

The (α, β, γ)-derivations of three-dimensional complex diassociative algebras are given as follows:

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}D$	Dim
$Dias_3^1$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	1
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	2
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	trivial	0
	(0,1,δ)	$\begin{pmatrix} -\delta d_{22} & 0 & 0 \\ 0 & -\delta d_{22} & 0 \\ 0 & 0 & -\delta d_{33} \end{pmatrix}$	2
$Dias_3^2$	(1,1,1)	$\begin{pmatrix} d_{11} & d_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	2
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	2
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	trivial	0
	(0,1,0)	trivial	0
	(0,1,δ)	$\begin{pmatrix} -\delta d_{22} & 0 & 0 \\ 0 & -\delta d_{22} & 0 \\ 0 & 0 & -\delta d_{33} \end{pmatrix}$	2

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}D$	Dim
$Dias_3^3$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	1
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	2
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	trivial	0
	(0,1,0)	trivial	0
	(0,1,δ)	$\begin{pmatrix} -\delta d_{22} & 0 & 0 \\ 0 & -\delta d_{22} & 0 \\ 0 & 0 & \delta^2 d_{22} \end{pmatrix}$	1
$Dias_3^4$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	2
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{22} & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	3
	(1,0,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & 0 & 0 \\ d_{31} & 0 & 0 \end{pmatrix}$	3
	(0,1,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,0,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	6
	(0,1,0)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,δ)	$\begin{pmatrix} t_9 & d_{12} & d_{13} \\ d_{21} & t_{10} & -d_{13} \\ 0 & 0 & -\delta d_{33} \end{pmatrix}$	4

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}D$	Dim
$Dias_3^5$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	2
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{11}d_{21} & 0 \\ 0 & 0 & t_1 \end{pmatrix}$	2
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,δ)	$\begin{pmatrix} d_{11} & 0 & 0 \\ t_{11} & -\delta d_{11} & 0 \\ 0 & 0 & -\delta d_{11} \end{pmatrix}$	1
$Dias_3^6$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{22} & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	3
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	trivial	0
	(0,1,0)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,δ)	$\begin{pmatrix} -\delta d_{11} & 0 & 0 \\ 0 & -\delta d_{11} & 0 \\ 0 & 0 & -\delta d_{11} \end{pmatrix}$	1

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}D$	Dim
$Dias_3^7$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & 0 \\ 0 & 0 & t_1 \end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & 0 \\ 0 & 0 & t_1 \end{pmatrix}$	2
	(1,0,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & 0 & 0 \\ d_{31} & 0 & 0 \end{pmatrix}$	3
	(0,1,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,0,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,δ)	$\begin{pmatrix} t_9 & d_{12} & d_{13} \\ d_{21} & t_{10} & -d_{13} \\ 0 & 0 & -\delta d_{33} \end{pmatrix}$	4
$Dias_3^8$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & d_{23} \\ 0 & 0 & d_{11} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & 0 \\ 0 & 0 & t_1 \end{pmatrix}$	2
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,δ)	$\begin{pmatrix} -\delta d_{33} & 0 & 0 \\ 0 & -\delta d_{33} & 0 \\ 0 & 0 & -\delta d_{33} \end{pmatrix}$	1

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}D$	Dim
$Dias_3^9$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & d_{23} \\ d_{31} & d_{31} & d_{33} \end{pmatrix}$	5
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	trivial	0
	(0,1,0)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	6
	(0,1,δ)	$\begin{pmatrix} -\delta d_{11} & 0 & 0 \\ 0 & -\delta d_{11} & 0 \\ 0 & 0 & -\delta d_{11} \end{pmatrix}$	1
$Dias_3^{10}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}$	3
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	3
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,δ)	$\begin{pmatrix} -\delta d_{11} & 0 & 0 \\ 0 & -\delta d_{11} & 0 \\ 0 & 0 & -\delta d_{11} \end{pmatrix}$	1

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}D$	Dim
$Dias_3^{11}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	4
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	3
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	trivial	0
	(0,1,0)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,δ)	$\begin{pmatrix} -\delta d_{11} & 0 & 0 \\ 0 & -\delta d_{11} & 0 \\ 0 & 0 & -\delta d_{11} \end{pmatrix}$	1
$Dias_3^{12}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}$	2
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	2
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	trivial	0
	(0,1,δ)	$\begin{pmatrix} -\delta d_{11} & 0 & -\delta d_{13} \\ 0 & -\delta d_{11} & 0 \\ 0 & 0 & -\delta d_{11} \end{pmatrix}$	2

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}D$	Dim
$Dias_3^{13}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	2
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	trivial	0
	(0,1,0)	trivial	0
	(0,1,δ)	$\begin{pmatrix} -\delta d_{11} & 0 & -\delta d_{13} \\ 0 & -\delta d_{11} & 0 \\ 0 & 0 & -\delta d_{11} \end{pmatrix}$	2
$Dias_3^{14}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	2
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{11} & d_{13} \\ 0 & 0 & d_{11} \end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	2
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	trivial	0
	(0,1,0)	trivial	0
	(0,1,δ)	$\begin{pmatrix} -\delta d_{11} & 0 & 0 \\ 0 & -\delta d_{11} & 0 \\ 0 & 0 & -\delta d_{11} \end{pmatrix}$	1

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}D$	Dim
$Dias_3^{15}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & 2d_{11} & 0 \\ 0 & 0 & 0 \end{pmatrix}$	2
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{11} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{11} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	3
	(1,0,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & 0 & 0 \\ d_{31} & 0 & 0 \end{pmatrix}$	3
	(0,1,1)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,0,1)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,δ)	$\begin{pmatrix} -\delta d_{11} & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & -\delta d_{33} \end{pmatrix}$	5
$Dias_3^{16}$	(1,1,1)	$\begin{pmatrix} t_2 & 0 & t_3 \\ d_{21} & t_4 & d_{23} \\ d_{31} & 0 & d_{33} \end{pmatrix}$ for $k=-1, r=\frac{p+n}{m}$ and $s=\frac{q}{m}$ $\forall m,n,p,q \in \mathbb{C}$	4
		$\begin{pmatrix} d_{33} & 0 & 0 \\ d_{21} & 2d_{33} & d_{23} \\ 0 & 0 & d_{33} \end{pmatrix}$ for $k \neq -1$, $m \neq 0$ and $q \neq 0$, $p,n \in \mathbb{C}$	3
		$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_5 & d_{23} \\ 0 & 0 & d_{33} \end{pmatrix}$ for $k \neq -1$, $m=0$, $q=0$ and $p,n \in \mathbb{C}$	4
		$\begin{pmatrix} d_{33} & 0 & 0 \\ d_{21} & 2d_{33} & d_{23} \\ 0 & 0 & d_{33} \end{pmatrix}$ for $k=1$ and $m=0$ and $q=0$,	3

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}D$	Dim
$Dias_3^{16}$	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{11} & d_{23} \\ d_{31} & 0 & d_{33} \end{pmatrix}$ for $k=0 \Rightarrow p=0, q=0;\ m,n \in \mathbb{C}$	5
		$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{11} & d_{23} \\ 0 & 0 & d_{33} \end{pmatrix}$ for $k=0 \Rightarrow p\neq 0, q\neq 0;\ m,n \in \mathbb{C}$	4
		$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{11} & d_{23} \\ 0 & 0 & d_{11} \end{pmatrix}$ for $k\neq 0$; for all $p,q,m,n \in \mathbb{C}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & d_{22} \end{pmatrix}$ for $k=0 \Rightarrow m=0, p=0;\ q,n \in \mathbb{C}$	5
		$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{11} & d_{23} \\ 0 & 0 & d_{11} \end{pmatrix}$ for $k=0 \Rightarrow m\neq 0, p\neq 0;\ q,n \in \mathbb{C}$	3
		$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{11} & d_{23} \\ 0 & 0 & d_{11} \end{pmatrix}$ for $k\neq 0 \Rightarrow \forall m,p,q,n \in \mathbb{C}$	3
	(1,0,0)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ d_{21} & 0 & d_{23} \\ d_{13} & 0 & d_{33} \end{pmatrix}$	6
	(0,1,1)	$\begin{pmatrix} d_{13}r & 0 & d_{13} \\ d_{21} & d_{22} & d_{23} \\ -d_{13}s & 0 & -d_{13}r \end{pmatrix}$ $m_1=-1, r=\frac{m_3+m_4}{2m_5}, s=\frac{m_2}{m_5}$	4
		$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & -d_{11} \end{pmatrix}$ for $m\neq -1$	4

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}D$	Dim
$Dias_3^{16}$	(0,0,1)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$ for $k=0$ at $p\neq 0$ and $m\neq 0$; $n,q\in\mathbb{C}$	3
		$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$ for $k=0$ at $p=0$ and $m=0$; $n,q\in\mathbb{C}$	6
		$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$ for $k\neq 0$ at $p,m,n,q\in\mathbb{C}$	3
	(0,1,0)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix}$ for $k=0,p=0,q=0,m,n\in\mathbb{C}$	6
		$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$ for $k=0p\neq 0,q\neq 0,m,n\in\mathbb{C}$	3
		$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$ for $k\neq 0,\forall p,q,m,n\in\mathbb{C}$	3
	(0,1,δ)	$\begin{pmatrix} -\delta d_{11} & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & -\delta d_{11} \end{pmatrix}$ for $k=0,\forall p,q,m,n\in\mathbb{C}$	4
		$\begin{pmatrix} t_{14} & 0 & -\delta k d_{13} \\ d_{21} & d_{22} & d_{23} \\ -\frac{\delta}{k}d_{31} & 0 & -\delta t_{14} \end{pmatrix}$ for $k\neq 0,\forall p,q,m,n\in\mathbb{C}$	5
$Dias_3^{17}$	(1,1,1)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & 0 & 0 \\ d_{31} & 2d_{21} & 0 \end{pmatrix}$	2
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{11} & 0 \\ d_{31} & d_{21} & d_{11} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{11} & 0 \\ d_{31} & d_{21} & d_{11} \end{pmatrix}$	3
	(1,0,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & 0 & 0 \\ d_{31} & 0 & 0 \end{pmatrix}$	3

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}D$	Dim
	(0,1,1)	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ d_{31} & d_{32} & d_{33} \end{pmatrix}$	3
	(0,0,1)	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ d_{31} & d_{32} & d_{33} \end{pmatrix}$	3
	(0,1,0)	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ d_{31} & d_{32} & d_{33} \end{pmatrix}$	3
	(0,1,δ)	$\begin{pmatrix} -\delta d_{11} & 0 & 0 \\ -\delta d_{21} & -\delta d_{11} & 0 \\ d_{31} & d_{32} & d_{33} \end{pmatrix}$	5
$Dias_3^{18}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	2
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{22} & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	3
	(1,0,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & 0 & 0 \\ d_{31} & 0 & 0 \end{pmatrix}$	3
	(0,1,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,0,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	6
	(0,1,δ)	$\begin{pmatrix} t_{12} & d_{12} & d_{13} \\ -d_{12} & d_{22} & -d_{13} \\ 0 & 0 & t_{13} \end{pmatrix}$	5

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}D$	Dim
$Dias_3^{19}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & d_{22} & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	trivial	0
	(0,1,δ)	$\begin{pmatrix} -\delta d_{33} & 0 & 0 \\ 0 & -\delta d_{33} & 0 \\ 0 & 0 & -\delta d_{33} \end{pmatrix}$	
$Dias_3^{20}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & 0 \\ 0 & 0 & t_1 \end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & d_{23} \\ 0 & 0 & d_{11} \end{pmatrix}$	3
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ -d_{11} & -d_{12} & -d_{13} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,δ)	trivial	0

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}D$	Dim
$Dias_3^{21}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ d_{21} & t_1 & d_{23} \\ d_{31} & d_{31} & d_{33} \end{pmatrix}$	5
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	6
	(0,1,0)	trivial	0
	(0,1,δ)	$\begin{pmatrix} -\delta d_{33} & 0 & 0 \\ 0 & -\delta d_{33} & 0 \\ 0 & 0 & -\delta d_{33} \end{pmatrix}$	1
$Dias_3^{22}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}$	3
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	2
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,δ)	$\begin{pmatrix} -\delta d_{33} & 0 & 0 \\ 0 & -(1/\delta)d_{33} & 0 \\ 0 & 0 & -\delta d_{33} \end{pmatrix}$	1

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}D$	Dim
$Dias_3^{23}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	4
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{22} \end{pmatrix}$	3
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	$\begin{pmatrix} d_{11} & d_{12} & d_{13} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,0)	trivial	0
	(0,1,δ)	$\begin{pmatrix} -\delta d_{33} & 0 & 0 \\ 0 & -\delta d_{33} & 0 \\ 0 & 0 & -\delta d_{33} \end{pmatrix}$	1
$Dias_3^{24}$	(1,1,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}$	2
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & d_{13} \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	3
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	trivial	0
	(0,1,0)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,δ)	$\begin{pmatrix} -\delta d_{11} & 0 & 0 \\ 0 & -\delta d_{11} & 0 \\ 0 & 0 & -\delta d_{11} \end{pmatrix}$	1

IC	(α,β,γ)	$Der_{(\alpha,\beta,\gamma)}D$	Dim
$Dias_3^{25}$	(1,1,1)	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}$	1
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	2
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{22} & 0 \\ 0 & 0 & d_{33} \end{pmatrix}$	3
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	trivial	0
	(0,1,0)	$\begin{pmatrix} 0 & 0 & 0 \\ d_{21} & d_{22} & d_{23} \\ 0 & 0 & 0 \end{pmatrix}$	3
	(0,1,δ)	$\begin{pmatrix} -\delta d_{11} & 0 & 0 \\ 0 & -(1/\delta)d_{11} & 0 \\ 0 & 0 & -\delta d_{33} \end{pmatrix}$	2
$Dias_3^{26}$	(1,1,1)	$\begin{pmatrix} t_6 & d_{12} & d_{13} \\ d_{21} & t_7 & d_{23} \\ d_{31} & d_{32} & t_8 \end{pmatrix}$	6
	(1,1,0)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1
	(1,0,1)	$\begin{pmatrix} d_{11} & 0 & 0 \\ 0 & d_{11} & 0 \\ 0 & 0 & d_{11} \end{pmatrix}$	1
	(1,0,0)	trivial	0
	(0,1,1)	trivial	0
	(0,0,1)	trivial	0
	(0,1,0)	trivial	0
	(0,1,δ)	$\begin{pmatrix} -\delta d_{11} & 0 & 0 \\ 0 & -(1/\delta)d_{11} & 0 \\ 0 & 0 & -\delta d_{11} \end{pmatrix}$	1

In the tables above the following notations are used.:

- IC: isomorphism classes of algebras.
- Dim: Dimensions of the algebra of derivations.
- $t_1 = d_{11} + d_{21}, \quad t_2 = d_{33} - rd_{31}, \quad t_3 = \frac{-q}{m} d_{31},$
- $t_4 = 2d_{33} - rd_{31}, \quad t_5 = d_{11} + d_{33}, \quad t_6 = -d_{21} - d_{31},$
- $t_7 = -d_{12} - d_{32}, \quad t_8 = -d_{13} - d_{23}, \quad t_9 = -\delta d_{33} - d_{21},$
- $t_{10} = -\delta d_{33} - d_{12}, \quad t_{11} = (-\delta - 1)d_{11} \quad t_{12} = d_{12} + d_{22} - d_{21},$
- $t_{13} = -\delta(d_{12} + d_{22}), \quad t_{14} = \frac{-\delta n - p}{m(1+\delta)}.$

4. Conclusion

As results we obtained the dimensions of $Der_{\alpha,\beta,\gamma}D$ for two-dimensional complex diassociative algebras vary between zero and two. The dimensions of the derivation algebras in three-dimensional complex diassociative algebras case vary between zero and six.

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