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Images and Metaphors of Mathematics Among University Students

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Abstract

This paper discusses the images and metaphors of mathematics held by university students through their experiences of learning mathematics and the application of mathematical knowledge in real life. The study involved two hundred and four university students selected using purposive sampling and data was obtained through a question form shared online. The analysis was based on descriptive statistics, namely comparing percentages among the responses, since the measurement level of the data collected was either categorical or ordinal and there was no continuous data collected. The findings indicate that images and metaphors related to mathematics held by university students focused more on physical aspects rather than non-physical. Thus, the lack of understanding and imbalance in mathematics learning involving metaphysical domains needs to be observed in detail.

Keywords: images; mathematics; metaphors; university students.

1 Introduction

Educators have different views about mathematical knowledge. According to one view, mathematical knowledge is regarded as an idea built by an individual based on their own experiences, and not as symbols on paper or a set of objects in empirical realms [12]. The fundamental point underlying this view is that the process of mathematical knowledge involves the development of mathematical knowledge, and the effort is operative. Furthermore, the quest for mathematical knowledge is full of value and is influenced by cognitive, affective, and metaphysical factors such as beliefs, conceptions, attitudes, motivation, feelings, and emotions. In-depth research on affective factors and the role of affect in mathematics education conducted by [10] shows that belief systems affect cognition, although some individuals may not be aware of their beliefs. However, Thompson [17] explains that the theoretical discussions about beliefs and belief systems in mathematics education have not been made entirely. The differences between beliefs and knowledge can hardly be explained. Furthermore, over the past decade, discussions and research on student images and metaphors related to mathematics and mathematical learning have become an important focus in mathematics education.

The purpose of this study is to identify the beliefs and subjective thoughts that university students have about mathematics and the mathematical learning experiences that they have. Such beliefs and thoughts are represented in the form of certain images and metaphors. This study uses a survey study design with a written question form technique and a purposive sampling method. Images and metaphors owned by university students about mathematics and mathematical learning involving circles of understanding are interconnected with each other. The findings of the images and metaphors of university students can provide a glimpse of the development of their conceptions and beliefs about mathematics. Hence, the study of these images and metaphors can serve as a component of epistemological knowledge of mathematics for the development of higher education among university students.

2 Literature Review

This study aims to identify the images and metaphors of university students on mathematics. Recent studies related to images and metaphors related to mathematics were mostly carried out to assist in providing information on how to learn mathematics, the way mathematics was taught to students, and how certain phenomena influenced the teaching and learning of mathematics.

Saban and Kocbeker [15] assert that metaphors can be used as a pedagogical tool to help students comprehend and describe abstract and complex concepts using much simpler and more understandable language. Educators can use metaphor analysis to assist prospective teachers in determining their teaching and learning values, beliefs, and philosophies. Saban and Kocbeker [15] discovered that the sun, sculptor, parent, compass, lighthouse, gardener, candle, tree, painter, and tour guide are the most frequently used metaphors to describe the concept of 'teacher' given by prospective teachers.

A study by Erdogan et al. [5] found out that among the top five metaphors to explain the concept of 'Mathematics' given by the prospective teachers are water, puzzle, ocean, sea and riddle. More than three quarter of the prospective teachers perceived Mathematics as 'limitless', 'interconnected', 'basically needed', 'fun', 'cumulative', 'indispensable' and 'foundation of other sciences'. The study suggests that the prospective teachers with positive opinion about mathematics can teach students the Mathematics subject in a professional way. While for the prospective teachers

with negative opinion given about Mathematics, an integration between mathematics, history and nature need to be incorporated into the curriculum in order to improve the negative attitude.

Yee [19] stated that metaphors are regularly used by mathematics teachers to relate difficult or complex concepts. The study investigated students' and teachers' metaphors about 'Mathematics problem solving' and found out the most popular metaphors are problem solving is a journey, discovery, searching, building, visualization, process, and partitioning. Meanwhile, Davis et al. [4] also found out that mathematics teachers discussed knowledge, teaching and learning based on extensive transcripts, and focusing on images and metaphors.

Valdez and Saulo [18] studied Philippines students' metaphors for addition, subtraction, multiplication, and division. The study discovered that the four operations' metaphors resemble the popular food metaphor. The findings show that learners view mathematical operations as emotional rather than mechanical processes.

Latterell and Wilson [9] investigated the mathematics metaphor between elementary education and mathematics teaching majors. About half of elementary education majors saw mathematics as a roller coaster ride. A quarter of them compare mathematics to walking through a field of bombs. More than half of mathematics education majors see mathematics as a way of thinking. Several of them describe mathematics as an essential body of content for other disciplines. Their findings show that the two groups with different major courses use different mathematical metaphors.

Cekirdekci [3] recently examined primary school students' perceptions of mathematics lessons and discovered 64 distinct mathematical metaphors. Among the top five metaphorical perceptions of mathematics are mathematics as a game, mathematics as a brain game, mathematics as a book, mathematics as a world, and mathematics as a sun. Additionally, the study classifies the metaphors as mathematical knowledge, mathematical teaching principles, mathematical abilities, and affective characteristics toward mathematics.

Osman et al. [13] explored the mathematical images and metaphors of achievers and non-achievers in two secondary schools in Pahang, Malaysia. The study discovered that images and metaphors about mathematics for both groups of students are classified into two major contexts in their lives, which are school and their daily routine as teenagers. Mathematics lessons are depicted in both groups as images of a teacher in the classroom, students learning without a teacher, mathematical symbols, books and tools, and real-life applications. The majority of students in both groups perceived mathematics-related activities as solving a riddle, followed by doing magic and exercising. Kahrizsangi and Barwell [7] used Cameron et al.' s [1] categorization to investigate the metaphor of mathematicians' discussions in radio broadcasts. They discovered seven systematic metaphors of mathematics in the contexts of negative numbers, symmetry, pi, the Poincaré conjecture, and prime numbers: slog, system, discovery, building and constructing, key, visuospatial, and mysterious.

In other studies, Carter [2] analyzed the presentation of mathematics diagrams and introduced the notion of faithful representation based on images and metaphors. The things studied in the study of the images and metaphors of students and teachers regarding mathematics include effective elements such as attitudes, motivations, values, emotions, and beliefs; learning methods of mathematical operations; teacher education such as in-service training, education of trainee teachers, and professional development; and cultures such as gender, culture-wide comparisons, and the role of language in mathematical learning. Past studies related to the images and metaphors of mathematical learners involve multiple aspects, except for metaphysical aspects involving metaphysical elements. One obvious thing is that this study relies heavily on empirical experience and

rational domain thinking, and there is no explicit and prominent attempt to consider religious teachings in discussion and research on student mathematical conceptions and mathematical education.

Sam [16] defined the term 'image of mathematics'as a mental representation or vision of mathematics that is probably formed as a result of social experiences mediated by school, parents, classmates or the media. However, in this study, the term images refer to personal images of mathematics. Such images involve a personal representation or depiction of mathematics through the use of a specific type of representation. Cognitive and affective dimensions can be present in such representations. An individual's mathematical image may contain implicit elements that the individual is unaware of or may be displayed to the public without conscious consideration [11]. According to Inan [6], evaluating students' and teacher' perceptions of the images reflected in mathematics is important. Therefore, it would be helpful when you are engaging in mathematics with students.

The majority of research on mathematical images and metaphors has concentrated on material and physical aspects, with no consideration given to or discussion of metaphysical aspects. This study examines mathematical and metaphorical images associated with mathematics learning that are used by university students, while also considering the metaphysical aspect. It encompassed spiritual dimensions as well as cognitive and affective dimensions. Thus, in comparison to other secular perspectives, a holistic perspective, such as a universal integrated perspective, is deemed more appropriate for use as a framework theory in this study. A universal integrated perspective is a philosophical, psychological, and sociological analysis of humanity's religious beliefs and education in a holistic and integrated context consistent with Malaysia's stated National Philosophy of Education [12].

3 Methodology

This study employed an online questionnaire where data was collected through a Google Form. This method of data collection has its advantages, such as a faster response rate, cheaper, more convenient, and data is quickly analyzed. Purposive or selective sampling was used since there was no intention of making generalizations about the population based on the sample selected. The question form link was distributed to undergraduate students and shared for two weeks to give ample time for them to respond. The design of the online question form was adapted from research conducted by [14]. The online question form consisted of five parts, which were background information, things or situations related to mathematics, participants' opinion on statements given on the source of mathematics, participants' opinion on mathematics knowledge and the parable on learning mathematics. The validity of the items in the question form was verified by a very knowledgeable and qualified Professor of Mathematics Education. The reliability or external consistency of the questions was established by conducting a test-retest procedure on 25 participants. A correlation of 0.87 was obtained, indicating a high test-retest reliability. The analysis was based on descriptive statistics, namely comparing percentages among the responses, since the measurement level of the data collected was either categorical or ordinal and there was no continuous data collected.

4 Findings

The number of participants who responded to the online questionnaire was 204, where 66.2% were female and 33.8% were male university students. Around 97% of them were Malays, with an age ranging from 17 to 23 years old. Diploma and degree level students comprised 58% and 19% of the participants respectively, while 22.9% were at the pre-university level (Foundation). The majority of them were interested in mathematics (88.5%), 9.6% were neutral and only 1.9% indicated they were not interested in mathematics.

4.1 Metaphors Related to Mathematics

Participants were asked in the questionnaire their choice of what represents mathematics with respect to activity, profession, instrument and element. Most participants (78%) perceived solving riddles as a metaphor for doing mathematics activities (refer to Figure 1), followed by doing exercise (30.7%) and only 9.8% considered doing mathematics activities as performing religious activities or ibadah. Thus, mathematics activities were regarded by participants as strongly related to physical aspects but not so to metaphysical aspects.

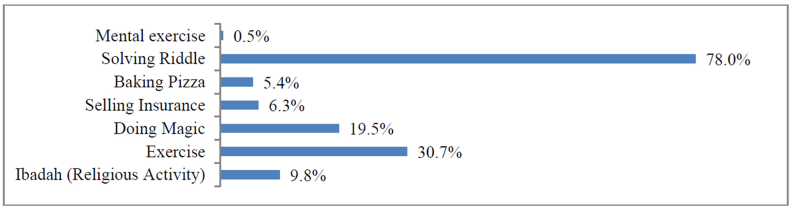

Figure 1: Activities related to mathematics.

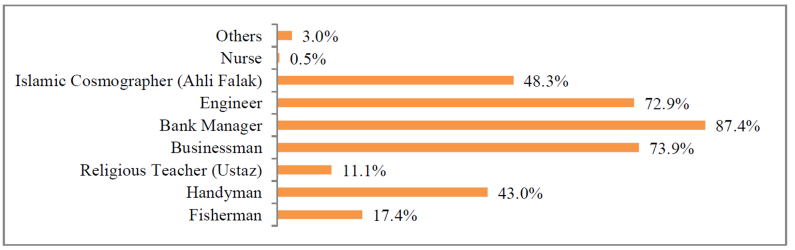

Figure 2: Professions related to mathematics.

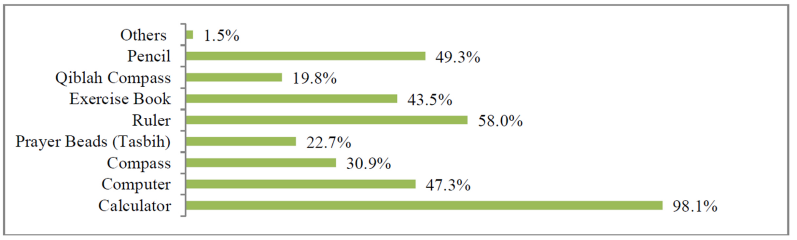

Figure 3: Instruments related to mathematics.

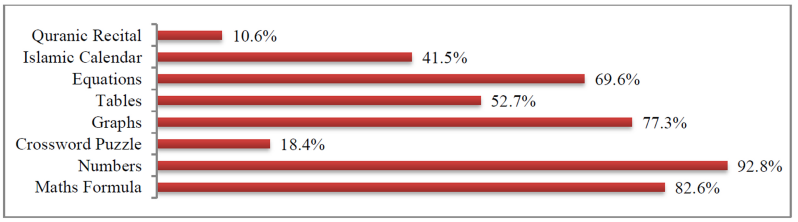

Figure 4: Elements related to mathematics.

For metaphors regarding profession; bank manager, businessman and engineer were the preferred choices, 87.4%, 73.9% and 72.9% respectively (refer to Figure 2) compared to a lowly 11.1% on religious teacher but a moderate 48.3% on Islamic cosmographer. For metaphors regarding instruments that were perceived to be related to mathematics, almost all participants (98.1%) chose calculator (refer to Figure 3). Metaphysical instruments such as prayer beads or qiblah compass gathered just 22.7% and 19.8% of participant's choices. Number and mathematical formulas (92.8% and 82.6% respectively) were the top choice by participants on metaphors that relate with mathematical elements (refer to Figure 4). Metaphysical elements such as Islamic calendar gathered 41.5% and a low 10.6% on Quranic recital. The findings obtained about physical aspects seemed to be consistent with the findings by [14].

4.2 Sources of Mathematics

The following four figures show the responses to four statements about the source of mathematics. The statements were as follows:

i) To me, knowledge of mathematics is part of the content of the universe and such knowledge is discovered by human through sensory experiences.

ii) To me, knowledge of mathematics is part of the content of the universe and such knowledge is inherent in the human thinking.

iii) To me, knowledge of mathematics comes from God and such knowledge is constructed by humans based on their thinking, sensory experiences and guidance from God.

iv) To me, knowledge of mathematics does not come from God but is constructed by man based on his thinking and specific experience only.

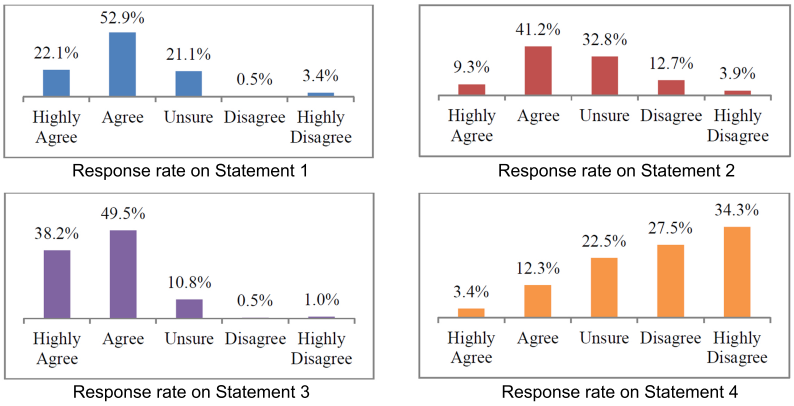

Figure 5: Response rate corresponding to Statements 1,2,3 and 4.

Based on Figure 5, majority of the participants seemed to be highly agree or agree on statement 3 (87.7%) which states that knowledge of mathematics comes from God and such knowledge is constructed by humans based on their thinking, sensory experiences and guidance from God. Statement 1 was agreed by 75% of the participants followed by statement 2 (50.5%). Only 15.7% of the participants agreed with statement 4 that knowledge of mathematics does not come from God but is constructed by man based on his thinking and specific experience only.

4.3 Nature of Mathematics

Participants' perception on the nature of mathematics was obtained through five items, namely, dynamism or the degree that mathematics changes, truthfulness, concreteness, origin and importance of mathematics. Based on Figure 6, 26.5% of participants considered mathematics to be changing sometimes while another 26.5% considered that there are not many changes in mathematics. More than half (61.8%) of the participants were in the opinion that mathematics is true in some situations, while less than half (25.5%) participants considered mathematics is true in all situations. In the aspect of concreteness, most participants (35.3%) considered mathematics may be reflected in a concrete way, while only 1% of the participants considered mathematics cannot be described in concrete terms. In terms of origin, 37.3% of the participants considered some parts of mathematics are created by human beings, while 27% of the participants considered mathematics as created totally by human beings. Finally, majority of participants (71.1%) considered mathematics as highly valuable, 24% valuable and only 0.5% viewed mathematics as not valuable.

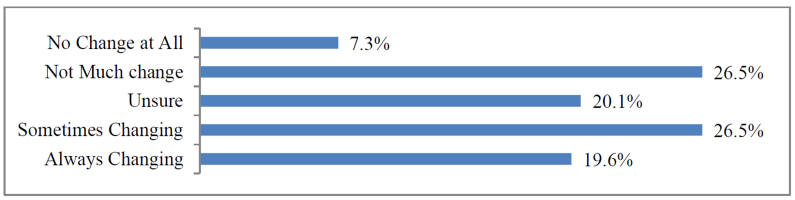

Figure 6: Perception toward dynamism of Maths.

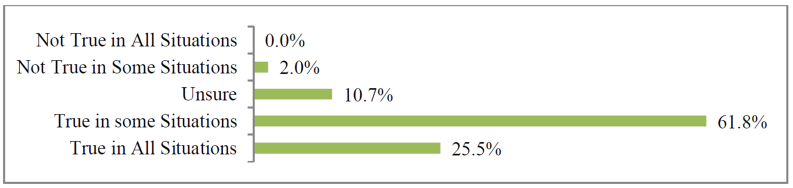

Figure 7: Perception toward truthfulness of Maths.

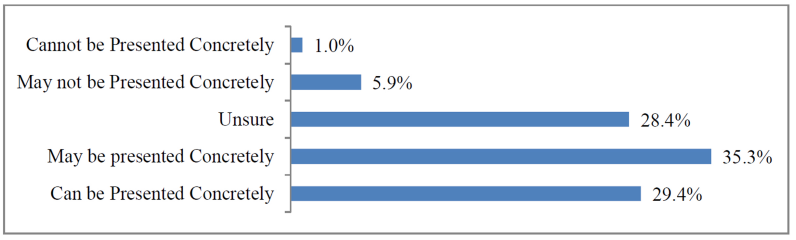

Figure 8: Perception toward concreteness of Maths.

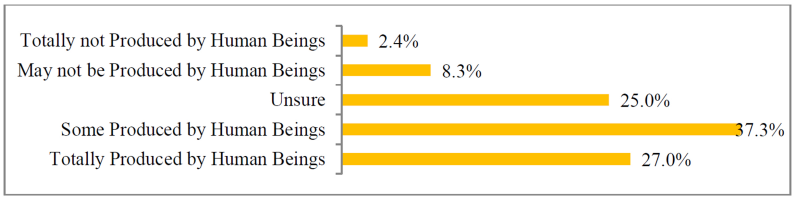

Figure 9: Perception toward origin of Maths.

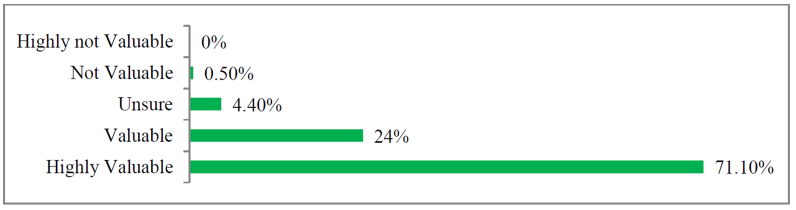

Figure 10: Perception toward importance of Maths.

4.4 Metaphors of Learning Mathematics

Participants responses in completing the sentence, 'Learning Mathematics is like .....' were analyzed by grouping them according to a common theme as suggested by [9]. Two of the themes were consistent with their findings which were 'Thinking Process' and 'A Struggle'. In this study, more than half of the participants (70.1%) viewed mathematics as a thinking process. Examples of statements that fall into this theme are as follows:

- Learning mathematics is like solving a jigsaw puzzle. Need to find the correct pieces and think about the whole picture of the jigsaw.

- Learning mathematics is like solving Sudoku. Need to think, strategize, focus and be patient. Feel real satisfaction if successful in solving it.
- Learning mathematics is like baking a cake. Need to get all the right ingredients to successfully bake the cake.
- Learning mathematics is like fishing. You need a lot of patience and good planning.

The next theme that emerged from the responses were that learning mathematics is a struggle (24%), indicating that mathematics is a difficult subject. Some statements that fall in this category are:

- Learning mathematics is like solving a friend's problem, so difficult.
- Learning mathematics is like climbing a mountain, need to think and exhausting.
- Learning mathematics is like a never-ending thing whenever you get an incorrect answer.
- Learning mathematics is like climbing the stairs. Once you fall at the bottom, you will never get to the top.

The next theme is unique in such a way that the statements relate learning mathematics to one's life and religious activity (Ibadah) (5.88%). Below are a few examples of such statements.

- Learning mathematics is like solving problems in life.
- Learning mathematics is like learning knowledge that are closely related to human's life since every of our actions relate to numbers, for example the time the action occurs.
- Learning mathematics is like searching for a solution to life problems that need patience and strategy.
- Learning mathematics is like doing ibadah or metaphysical activity that need to do often in order to understand better and easier.
- Learning mathematics is like to know the Creator of the universe.
- Learning mathematics is 'ilmu kifayah' and an act of worship to Allah.

The metaphors formed by participants to explain their role during mathematical learning can be classified into three categories: metaphors that satisfy some of the universal integrated perspective features, information processing, and social perspective (see Table 1). In this study, the dominant perspective and influence on most of the metaphors formed by the participants was the perspective of information processing. When a metaphor is classified as a representation with the features of a certain perspective, such as information processing, it does not mean that explicit beliefs and conceptions of information processing have been identified. On the other hand, an attitude, tendency, or set of values similar to an information processing perspective has been concluded. Furthermore, based on the student's explanation of the metaphor that they designed, there is an overlap between some of these explanations. In other words, the explanation contains a mix of features from two or more different perspectives. Therefore, there is a metaphor that does not entirely fulfill the characteristics of a single perspective. In this context, clarification is based on the perspective features that are more prominent than in other perspectives.

Table 1: Distribution of participants according to the category of metaphors.

Category of Metaphor	Frequency	Percentage (%)
Universal Integrated Perspective	12	5.88
Information Processing	143	70.1
Social Perspective	49	24.0

5 Conclusion

The findings of this study on metaphors related to mathematics, sources of mathematics, and the nature of mathematics are consistent with the results of previous research by [13]. The metaphors formed by participants to explain their role during mathematical learning can be classified into three categories: metaphors that satisfy some of the universal integrated perspective features, information processing, and social perspective (see Table 1). In this study, the dominant perspective and influence on most of the metaphors formed by the participants was the perspective of information processing. When a metaphor is classified as a representation with the features of a certain perspective, such as information processing, it does not mean that explicit beliefs and conceptions of information processing have been identified. On the other hand, an attitude, tendency, or set of values similar to an information processing perspective has been concluded. Furthermore, based on the student's explanation of the metaphor that they designed, there is an overlap between some of these explanations. In other words, the explanation contains a mix of features from two or more different perspectives. Therefore, there is a metaphor that does not entirely fulfill the characteristics of a single perspective. In this context, clarification is based on the perspective features that are more prominent than in other perspectives.

The development of an understanding of something involves raising awareness about the interconnectedness of certain things, internal experimentation, inward action, and intuition or inspiration. Everything can lead to a meaningful acquisition of something if the process is carried out in accordance with religious teachings (for example, the principles contained in the Absolute Frame of Reference). The images and metaphors owned by university students about their role as mathematical learners are related to their conceptions, beliefs, values, and their understanding of mathematics and the role of mathematical learners. If students have a holistic, integrated, balanced, and clear concept and understanding of mathematics and their role as mathematical learners, the choice of images and metaphors they describe in mathematics and mathematical learning will likely highlight constructive features. However, the dependence on names for the mathematical and metaphorical images used by university students to visualize mathematical learning can lead to conclusions that do not reflect the current situation. The findings show that there are situations in which university students have different explanations, although the meanings of the metaphors they use are the same. For example,

> Learning mathematics is like solving a friend's problem. It is so difficult. Learning mathematics is like solving problems in life.

In this case, the researcher needs to be careful when discussing and classifying the students' metaphors. They need to consider the explanations provided by the students about the metaphors they use and identify the names of the metaphors. This view is consistent with the assertion made by Kittay [8] that the metaphor has a specific cognitive meaning and is not merely a literal meaning. The metaphors of mathematics learning among the university students described above

are significantly classified as information processing. These metaphors can serve as a component of epistemological knowledge of mathematics for the development of higher education among university students.

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Conflicts of Interest The authors declare no conflict of interest.

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