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Students' Analogical Reasoning in Solving Trigonometric Target Problems

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Abstract

Analogical reasoning plays a crucial part in problem-solving since it requires students to connect prior knowledge with the issues at hand in learning mathematics. However, students struggle when developing solutions to the issues utilizing analogies even if there is a connection between mathematical creativity and analogical reasoning. The aims of this study were to assess students' use of Ruppert's phases to solve problems and identify students' analogy patterns to solve target problems. This study is qualitative in nature. Of 19 research participants, six were then chosen using the purposive sampling technique based on their levels of mathematical creative ability. Test, interview, and documentation were the data gathering techniques used in this study. The study's findings suggested that good analogical reasoning skills did not serve as a prerequisite for students with strong mathematical creative thinking skills. Only one subject out of three who possessed necessary mathematical creative thinking abilities could go through the four steps of analogical reasoning-structuring, mapping, applying, and verifying. All other subjects were unable to complete the four steps of analogy, and even their creative thinking skills were weak. This was because the students did not comprehend the idea and could not connect prior knowledge with the issues at hand. In order to remind students of their prior knowledge and experiences, it would therefore be necessary at this analogy stage to establish an initial stage before structuring. The format and degree of difficulty of the questions were assumed to be other elements that might influence students' responses. The results of this study are expected to be a reference for further research, namely increasing analogical reasoning optimally as an effort to increase students' prior knowledge and students' mathematical creative thinking abilities in solving mathematical problems.

Keywords: analogical reasoning; mathematical creative thinking; trigonometry.

1 Introduction

Mathematics has yet to become an interesting subject for most students, and sometimes it even ends up being one of the most terrifying to them [32]. Even though mathematics is a compulsory subject taught in every educational institution. Mathematics proficiency is crucial because this subject is the master of science [21]. Mathematics is needed in various fields, even in solving life problems. Mathematics concerns real concepts, relations, and operations, as well as abstract principles. Thus, to solve problems, it is not enough to only have a basic understanding; it will require a high level of thinking process. Solving mathematical problems cannot be divorced from the way individuals process their cognition. The process is linked to how they receive information, how they process information, and determine the outcome. However, the evidence indicates that numerous students merely memorize procedures for solving mathematical problems without comprehending the significance underlying these procedures [33].

Analogical reasoning is a cognitive process of establishing connections between representations, frequently between existing and new ones [30]. Analogies play a key role in higher-level cognition, and because of their pervasiveness and broad influence, it is crucial to understand how analogical reasoning develops in order to comprehend human cognition more broadly [11, 14]. When the answer to a current issue is discovered by drawing parallels between that issue and an earlier one, this is known as analogical reasoning. In particular, analogical reasoning involves drawing conclusions and formulating assumptions about a new situation (the so-called "target problem") using a previous, well-known situation (referred to as the "source analogy" or the "source problem") [10]. Analogical reasoning involves the use of adaptability, memory, and reasoning in order to solve difficult analogies [12]. According to [18], a problem-solving process involves two steps, namely, recognition, which is the process of relating target problems to source problems, and analogical reasoning. [18] also argued that identifying a primary problem is essential to resolving a target issue, where the source issue should be comparable to the target issue. To address the target problem, new completion steps must be implemented after resolving the cause problem.

In the present research, the process of resolving analogical problems used some analogical reasoning components, namely, structuring, mapping, applying, and verifying [26]. Structuring refers to determining each mathematical element present in the target issue by drawing conclusions from identical links between the source problem and the target problem by encoding the objects or properties. Finding identical character code linkages between the source problem and the target problem is known as mapping. After drawing conclusions from the similarity or identity of the character code relationships between the source problem and the target problem, the relationships are then mapped to the target problem. Applying means using the steps used to solve the source problem to solve the target problem. Finally, verifying entails examining the target problem's solution and determining whether it is appropriate with the source problem. This study analyzed the analogical reasoning of students in solving target problems. Throughout the process of solving these problems, two things were to be found:

1. the reasoning of students in solving problems following Ruppert's stages, where the stages they could and could not perform were identified,
2. the analogy patterns established by the students when solving the target problems,

which were also based on the mathematical creative thinking abilities of the students.

Reasoning and thinking together are imperative for students in order for them to be able to represent their ideas in solving problems. Thinking creatively is at the highest level. Divergent thinking is frequently used to define creative thinking. In an OECD-conducted PISA study, creative thinking is defined as the capacity to actively participate in the generation, assessment, and refinement of ideas, which can result in novel and efficient solutions, developing knowledge and expressions that affect imagination [4]. Additionally, creativity is acknowledged as being essential to the imaginative capacity of humans across all fields, and it is clear that its impact permeates all aspects of life [22]. According to a growing body of research in recent years [6, 27] and [34], creativity is a necessary 21st-century ability that can be fostered and should be incorporated into the curriculum from an early age. In general, analogy research has prioritized analogical transfer as a method for learning in development and education, as well as for analytical thinking and problem-solving, placing less emphasis on creativity [2, 13].

According to research, training can be used to teach, learn, and enhance creative thinking [28]. In a mathematical setting, problem-solving is typically connected with creative mathematical thinking, especially in open-ended inquiries [17, 29]. The fluency component looks at a student's active and accessible understanding of a particular mathematical activity. When tackling an open mathematics problem, the flexibility component looks at how quickly one can switch mental states (for instance, by using a different mathematical concept each time). Finally, the originality factor determines how distinctive the student's response is and whether it is the same as the majority of other responses [8].

Analogical problems have been used in research by some researchers. For instance, [9] and [7] used algebra and combinatorics analogical problems and analogical challenges involving statistical notions with combined difficulties and independent events, respectively. In this research, analogical problems related to trigonometry were used. Previous research on analogy problems in trigonometry has also been carried out by [5] with the aim of research to see students' analogical reasoning in solving trigonometric problems in terms of cognitive style. The results of this study concluded that when viewed from a systematic cognitive style, students were able to understand the problems given, use information to solve problems, and apply their own methods to solve problems in a structured manner, while students with intuitive cognitive styles were able to understand problems and use information well, but in solving the problem is not yet structured. Students with an intuitive cognitive style prefer trial and error, rely on cues, and easily jump from one solution to another and students with a systematic cognitive style tend to analyze and interpret problems systematically, carefully, and carefully by making careful planning before starting. settlement process. Thus, it was found that there were differences in previous research and research conducted by this researcher, among others, in this study, the researcher reviewed analogical reasoning based on students' creative thinking abilities.

The purpose of this study was to assess the stages of analogical reasoning carried out by students in solving target problems and identify patterns of reasoning carried out by students, especially in trigonometry problems. Because trigonometry is one of the material requirements that must be mastered by students before studying and solving calculus problems. So far, what has happened in learning mathematics in tertiary institutions, especially students in mathematics education, is the lack of initial knowledge so that they will continue to experience difficulties in learning and subsequent mathematical problems. Analogical reasoning is very important to do in the Higher Education mathematics learning environment to be able to develop their reasoning abilities by remembering the initial concepts that have been previously studied and then applied to the problem to be solved. However, in analogical reasoning is still very rarely done in learning mathematics, even though analogical reasoning can also develop students' creative thinking skills through its stages, namely when digging up initial knowledge information, applying it to the problem to be solved, and verifying answers.

2 Methodology

This study was to analyze the analogical reasoning of students in solving problems following Ruppert's stages of structuring, mapping, applying, and verifying. It also attempted to find the analogy patterns established by the students when solving target problems. This analysis was based on the levels of mathematical creative thinking ability of the students: high, moderate, and low.

Table 1: Levels of creative thinking ability.

Criteria	Score
High	≥ 67
Moderate	$33 - 67$
Low	≤ 33

A descriptive-qualitative approach is used in this study. The population of this study were all even semester students who were taking trigonometry courses for the 2020/2021 academic year, totaling 19 people (n=19). All these students were given a creative thinking ability test and then six people were selected based on the score and level of mathematical creative thinking ability, namely for a high creative level by using interval of score ≥ 67, for a moderate creative level $33 <$ score < 67 and a score for a low creative level ≤ 33 [3]. The grouping of high, moderate, and low creative thinking abilities is explained in Table 2. The six students were each coded AL, LF, PU, NH, ZH, and TM. The selection of these six samples was also based on purposive sampling, namely a sampling technique from data sources based on certain considerations [31]. The considerations given are those that make the most mistakes.

Table 2: Research subjects.

No	Code	Level of mathematical creative thinking ability
1	AL	Moderate
2	LF	
3	PU	
4	NH	Low
5	ZH	
6	TM	

In this study, no students were with high mathematical creative thinking ability. Thus, only students with moderate and low mathematical creative thinking ability were involved. The instrument used in this study was a test composed of target problems regarding trigonometric equations. The validation of the instrument was carried out by two experts, according to whose suggestions the instrument was revised. Additionally, a reliability test was also carried out on the instrument. Finally, the instrument was used to test the students' analogical reasoning ability.

The data collection techniques employed in this study consisted of test, questionnaire survey, and in-depth interview. Test was used to determine students' problem-solving abilities according to Ruppert's analogical reasoning steps. Interview was used to determine the depth of students' answers in solving the functional problems presented to them with special attention to the analogical reasoning steps. The data collected were analyzed following the steps of data reduction, data presentation, and conclusion drawing. Based on the data obtained from the research subjects' test results, interviews were carried out to confirm the research subjects' work.

Data reduction was conducted to allow for an eventual drawing of acceptable and accountable conclusions. After reduction, data were presented in a descriptive form. The last step was drawing conclusions, which constituted an important activity in data analysis. After establishing and verifying assumptions based on the data and information collected, several conclusions were drawn.

3 Finding and Discussion

A summary of the achievements of the analogical reasoning steps of the six students, who were the research subjects and sources of data in this study, is presented in Table 3.

3.1 The mathematical analogical reasoning of students with moderate mathematical creative thinking ability in the structuring and mapping stages

Not all students with mathematical creative thinking skills were able to solve trigonometric equation problems well. At the structuring stage, it would be very important for students to remember the concepts that they had learned previously. However, students still found difficulties in using the concepts that they had learned and in identifying the source problem that they would use to solve the target problem. For example, the students were presented with the following problem: $4\sin^2(x) - 4\sin(x) = 3$ and were asked to find the solution to it. The following is an answer given by PU:

$$\begin{aligned}4\sin^2(x) - 4\sin(x) &= 3,\\ 4\sin(x)\big(\sin(x) - 1\big) &= 3,\\ 4\sin(x)\big(\sin(x) - 1\big) - 3 &= 0.\end{aligned}$$

Based on this answer, PU failed in the structuring stage and therefore could not proceed to solve the problem. During an interview, PU expressed his confusion about where to start solving the question. He was not able to recall the concept of factoring. Therefore, he could not finish the following stage, which was mapping, either. The following is an excerpt of the interview:

Table 3: Students' mathematical analogical reasoning steps achievements.

Step	Indicator	Subject					
		AL	LF	PU	NH	TM	ZH
Structuring: Identify the form of each equation existing in the source problem by coding the source problem's attributes or characteristics, and conclude identical relationships in the code of the source problem.	1. Be able to give examples of linear and quadratic equations in the source problem	✓	✓	✓	✓	✓	✓
	2. Identify the trigonometric identity in the source problem which will then be applied to the target problem	✗	✓	✓	✗	✗	✗
	3. Identify the numerator and denominator in the source problem to solve the target problem	✗	✓	✓	✗	✗	✗
Mapping: Look for identical relationship between the source problem and the target problem and draw conclusions from the similarity/identical relationship between the source problem and the target problem.	1. Find the displacement rule in linear equations in the source problem and the target problem	✓	✓	✓	✓	✓	✓
	2. Find the factoring in the quadratic equations in the source problem and the target problem	✗	✓	✗	✗	✗	✓
Applying: Apply the conclusions from the source problem to the target problem to solve the target problem	Apply the steps in solving the source problem regarding the following rules:						
	1. Moving segments	✓	✓	✓	✓	✓	✓
	2. Factoring	✗	✓	✗	✗	✗	✓
	3. Using numerators and denominators	✗	✓	✓	✗	✗	✓
	4. Using trigonometric identities for target problems	✗	✓	✓	✗	✗	✗
Verifying: Re-check the correctness of the solution of the target problem by checking the agreement between the target problem and the source problem	Solve the target problem by performing calculations using the concept or method applied in solving the source problem	✗	✓	✓	✗	✗	✗

Q : How did you end up with that answer?
PU : I apologize, I did not know how to solve the problem.
Q : Have you learned factoring?
PU : I have, Ma'am, during high school and college.
Q : Did it not occur to you that the question is like a quadratic equation? Please look at it again.
PU : (Looking back at the answer). I am confused, Ma'am.
Q : Well, the trigonometric equation has this form: $4\sin^2(x) - 4\sin(x) = 3$. If we make an analogy of it in a quadratic equation (source problem), we can write it as $4x^2 - 4x - 3 = 0$, where x can be substituted for $\sin(x)$.
PU : Oh, yes, Ma'am. I did not think of it that way at all.
Q : Why didn't you think of it that way?
PU : Maybe because the first thing that occurred to my mind when I saw the problem was that it was difficult. But then I realized that it was not as difficult as I thought. If that is how it should be done, then I can do factoring and solve the problem, Ma'am.

Another student, AL, was able to complete the problem well. He could identify the source problem by recognizing the target problem as a form of ordinary quadratic equation. He could also perform factoring well by converting $4\sin^2(x) - 4\sin(x) - 3 = 0$ into $\big(2\sin(x) - 3\big)\big(2\sin(x) + 1\big)$. He could perform the structuring stage by recognizing the concept of quadratic equations, but he made some factoring errors in the mapping stage. The following is an excerpt of the interview where he confirmed his answer:

Q : Do you know about factoring?
AL : Yes, Ma'am. I do.
Q : If factoring is performed on the equation $4A^2 - 4A - 3 = 0$, then what is the result?
AL : The result is $(2A + 3)(2A - 1)$, Ma'am.
Q : Are you sure?
AL : Very sure, Ma'am.
Q : Okay then. Try the reverse operation with $(2A+3)(2A-1)$. Do you get $4A^2 - 4A - 3 = 0$ too?
AL : Okay, Ma'am. I write $(2A + 3)(2A - 1) = (2A)(2A) - (2A)(1) + (3)(2A) - 3 = 4A2 - 2A + 6A - 3 = 4A2 - 4A - 3$. Here is the result, Ma'am.
Q : Are you sure it is $-2A + 6A = -4A$?
AL : Yes, ma'am.

Based on the interview excerpt above, the student still did not understand the concept of adding integers. For example, he could not perform operations when a negative integer met a positive integer correctly. It was also figured out that the student had difficulties in 1) writing the addition of integers and drawing a number line, 2) understanding the concept of subtraction and mixed-integer operations, and 3) determining the results of subtracting integers as he was not being thorough and did not understand the meaning. Indeed, the material on integer operations is difficult for students to understand, and the same holds true even for college students. One of the reasons for this is that students did not understand the basic concepts on this topic when they were in elementary school. Other reasons include, among other things, motivation, learning interest, teaching materials, and the ability level of the student. Difficulties related to operations and principles are most frequently experienced by students (79.4%) [19].

Other findings were found from the students' solutions to the problem

$$2\cos(x)\cot(x) - \sqrt{2}\cot(x) = 0.$$

LF was able to identify the question. He could change the form of the question using the trigonometric identity $\cot(x) = \dfrac{\cos(x)}{\sin(x)}$ in order to map the target problem. Still using the trigonometric identity, LF was also able to restructure the equation $2\cos^2(x) - \sqrt{2}\cos(x) = 0$ into the quadratic equation $2a^2 - \sqrt{2}a = 0$, which was then mapped to the target problem in order to derive a solution from the equation.

Based on students' answers to the problem $2\cos(x)\cot(x) - \sqrt{2}\cot(x) = 0$, the flexibility of each student in answering the problem was identified. There was a difference in how LF and AL answered this problem. LF used mapping by assuming $\cos(x)$ as a, while AL neither used mapping by assuming $\cos(x)$ nor used a quadratic equation. Instead, AL applied mapping by moving equations. The following is a comparison of how LF and AL solved the problem:

Table 4: Comparison of LF's and AL's answers.

Problem: $2\cos(x)\cot(x) = \sqrt{2}\cot(x) = 0$	
LF's Answer	AL's Answer
$2\cos(x)\cot(x) - \sqrt{2}\cot(x) = 0$ $2\cos(x)\dfrac{\cos(x)}{\sin(x)} - \sqrt{2}\dfrac{\cos(x)}{\sin(x)} = 0$ $2\dfrac{\cos^2(x)}{\sin(x)} - \sqrt{2}\dfrac{\cos(x)}{\sin(x)} = 0$ $2\cos(x)\dfrac{\cos(x)}{\sin(x)} = \sqrt{2}\dfrac{\cos(x)}{\sin(x)}$ → identify fractions (have the same denominator) Performing mapping: Make a distinction of $\cos(x) = a$: $\dfrac{2a^2 - \sqrt{2}a}{\sin(x)} = 0 \rightarrow$ $2a^2 - \sqrt{2}\,a = 0$ $(2a - \sqrt{2})(a) = 0$	$2\cos(x)\cot(x) - \sqrt{2}\cot(x) = 0$ $2\cos(x)\dfrac{\cos(x)}{\sin(x)} - \sqrt{2}\dfrac{\cos(x)}{\sin(x)} = 0$ $2\cos(x)\dfrac{\cos(x)}{\sin(x)} = \sqrt{2}\dfrac{\cos(x)}{\sin(x)}$ $2\cos(x)\dfrac{\cos(x)}{\sin(x)} = \sqrt{2}\dfrac{\cos(x)}{\sin(x)}$ $2\cos(x) = \sqrt{2}\dfrac{\cos(x)}{\sin(x)}\dfrac{\sin(x)}{\cos(x)}$ $\cos(x) = \dfrac{\sqrt{2}}{2}$

The difference between students in structuring and mapping reflected the creativity of each student in arriving at the same and correct answer. LF's answer was closer to the actual answer. The problem $2\cos(x)\cot(x) - \sqrt{2}\cot(x) = 0$ does not only have one solution. The problem is an open problem without any limit as to the value of $\cos(x)$, so the angle can be anywhere from $0°$ to $360°$.

In answering the problem $2\sin(2x) - 3\sin(x) = 0$, LF was also able to perform structuring and mapping well. LF was able to use trigonometric identities to identify the source problem using $\sin(2x) = 2\sin(x)\cos(x)$. Meanwhile, AL performed mapping by using $2A = 2\sin(A)$ when it should have been $\sin(2A) = 2\sin(A)\cos(A)$. Thus, at the structuring and mapping stages, even though the students had sufficient quality for creative thinking, it turned out that errors were still found. Errors were found in structuring and mapping the source problem. This could be caused by students' inaccuracies, forgetting factors, and lack of conceptual understanding. The ability to correctly differentiate between a variety of potentially relevant source analogies when tackling new problems is a requirement for mathematical proficiency. Experimental results in two different mathematical contexts demonstrated that introducing cues to support comparative reasoning during an initial instructional analogy resulted in an improved ability to distinguish between pertinent analogies at a later test. This is in contrast to teaching the same analogy and solution strategies without such cues [25].

3.2 The mathematical analogical reasoning of students with low mathematical creative thinking ability in the structuring and mapping stages

In solving trigonometric problems, students with low creative thinking skills were not able to carry out the problem-solving process properly. NH and ZH could only answer two of the four questions given. NH looked hesitant in answering the following problem:

$$4\sin^2(x) - 4\sin(x) - 3 = 0.$$

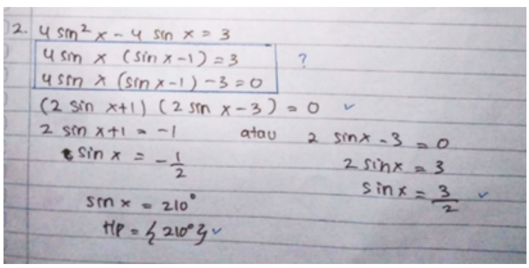

Figure 1: NH's answer in the structuring and mapping stages.

As shown in Figure 1, NH appeared unsure of using her answer to solve the problem. This can be seen from the first and second lines in NH's answer, which read:

$$4\sin(x)\big(\sin(x) - 1\big) = 3,$$
$$4\sin(x)\big(\sin(x) - 1\big) - 3 = 0.$$

These lines were then followed by factoring. However, the factoring process carried out by NH was correct. This led the researcher to ask her in person through an interview to confirm the answer. It turned out that when structuring the target problem, NH was confused and unable to solve it. In the end, NH admitted that she copied her friend's answer. Meanwhile, there were no errors found in ZH's answer. From his answer, ZH looked capable of structuring and mapping problems and even demonstrative of flexibility and originality in thinking. As shown in Figure 2, ZH provided an additional explanation of the completion of the answer, which was not necessary.

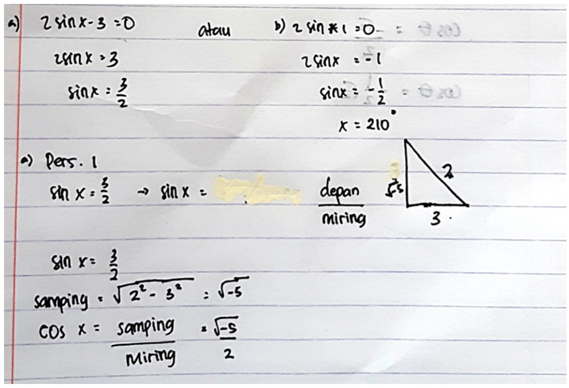

Figure 2: ZH's answer in the structuring and mapping stages.

The next answer to be discussed is that by TM. In solving a trigonometric equation, TM could perform structuring and mapping well by assuming $\sin(x) = a$. However, he performed factoring incorrectly. Meanwhile, in answering the problem $2\sin(2x) - 3\sin(x) = 0$, TM was unable to carry out structuring well. By assuming $\sin(x) = a$, TM made mistakes in using the trigonometric identity $\sin(2x)$. TM assumed that $\sin(2x) = 2\sin(x)$, so he wrote the example $\sin(x) = a$ as shown in Figure 3:

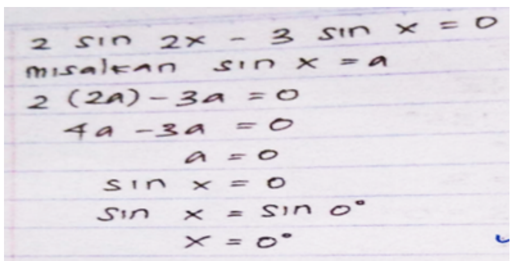

Figure 3: TM's answer in the structuring and mapping stages.

To the next problem $2\cos x \cot x - \sqrt{2}\cot x = 0$, alike to ZH and NH, TM could not answer either.

3.3 The analytical mathematical reasoning of students with moderate mathematical creative thinking ability in the applying and verifying stages

In the applying and verifying stages, students were asked to apply the conclusions from the source problem to the target problem. In this stage, students could solve the target problem and re-examine the correctness of the target problem solution. Students with moderate mathematical creative thinking skills could also apply the conclusions they had derived, but they were often not careful in using concepts and performing calculations, which caused errors in their answers. Contrarily, the students were not as good in the verifying stage. When arriving at an answer, the students did not re-check the target problem against the source problem. Students often made simple answers and were hesitant to think further to check the agreement between the target and source problems. In addition, some students were also found to work on the questions only halfway to the end, as was shown by AL in the following:

$$\sin(2x) - 3\sin(x) = 0.$$

For example:

$$\begin{aligned} \sin(x) &= A, \\ 2(2A) - 3A &= 0, \\ 4A - 3A &= 0, \\ A &= 0, \\ \sin(x) &= 0, \\ \sin(x) &= \sin(0). \end{aligned}$$

In this answer, students did not make any further explanation as to $A = \sin(x)$ and the value of x as the solution of the equation. This was because the students were not accustomed to using verbal language in solving math problems. Consequently, the researchers found it difficult to understand what the students intended to explain. Based on interviews, it was found that the students themselves did not understand the problems that they were going to solve. Students often did not think and work hard enough in solving problems. They worked on problems perfunctorily. In the next problem, AL also did not verify the form of the quadratic equation, which should be made equal to 0 first before being factored. After factoring, it turned out that AL left out the verifying or re-checking step. For instance, he only went as far as writing $\sin(x) = -\frac{3}{2}$, while he was supposed to calculate the size of the angle x. Likewise, from his answer $\sin(x) = \frac{1}{2} = 30^0$, it appears that AL could not differentiate between the value of $\sin(x)$ and the size of the angle x.

Table 5: AL's answer.

Problem: $4\sin^2(x) - 4\sin(x) - 3 = 0$		
Example:		
$\sin(x) = A$		
$4A2 - 4A - 3 = 0$		
$(2A+3)(2A-1)$	$\rightarrow$ Not made equal to zero	
$2A + 3 = 0$	$2A - 1 = 0$	
$2A = -3$	$2A = 1$	
$A = -\frac{3}{2}$	$A = \frac{1}{2}$	
$\sin(x) = -\frac{3}{2}$	$\sin(x) = \frac{1}{2} = 30^\circ$	$\rightarrow$ Still wrong in verifying answer
Should be $\rightarrow$	$x = 30^\circ$	

On the other hand, LF could solve the trigonometric equation problems. LF could demonstrate a quite flexible thinking process by offering more than one interpretation of the problems. To the problem $4\sin^2(x) - 4\sin(x) - 3 = 0$, LF offered two answers, namely $x = 210^\circ$ and $x = 330^\circ$, in which case x was written as a solution set. In the problem to which students were given the freedom to give more than one answer, it was not stated whether the angle should be between 0° and 90° $(0^\circ \leq x \leq 90^\circ)$ or between 0° and 360° $(0^\circ \leq x \leq 360^\circ)$.

3.4 The analytical mathematical reasoning of students with low mathematical creative thinking ability in the applying and verifying stages

Students with low mathematical creative thinking skills were not able to complete the applying and verifying stages, as shown by the answers of the three students, NH, ZH, and TM. NH was not able to solve problems and admitted that she copied her friends' answers. Meanwhile, ZH had performed verification, but it was not in accordance with what was expected in the question. Likewise, TM was not able to apply the two stages to the source problems. TM's knowledge and conceptual understanding about quadratic equations were still weak, so he failed to solve the trigonometric problems completely. The following is TM's answer to the problem $4\sin^2(x) - 4\sin(x) - 3 = 0$.

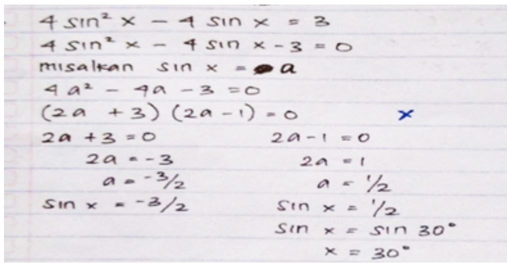

Figure 4: TM's answer in the applying and verifying stages.

In order to create something new, one needs to combine divergent and rational thinking into creative thinking. One sign of mathematical creativity is the creation of something novel, whereas the other signs pertain to logical reasoning and divergent thinking [24]. Based on level of creative thinking abilities, the subjects in this study were divided into students with moderate mathematical creative thinking ability and students with low mathematical creative thinking ability. Of the 19 second-semester students in the 2020/2021 academic year who were given trigonometric equation problems, four students or 21.05% of all students were in the category of students with moderate mathematical creative thinking ability, fifteen students or 78.95% of all students were in the category of students with low mathematical creative thinking ability, and no one was in the category of students with high mathematical creative thinking ability. Due to the absence of students with high mathematical creative thinking ability, the subjects selected in this study were representatives of students with moderate and low mathematical creative thinking ability only. There were six subjects selected, who were coded by their initials as AL, LF, PU, NH, TM, and ZH.

Based on Table 3, the student with moderate mathematical creative thinking ability, LF, had been able to use trigonometric identities at the structuring stage. Meanwhile, PU and AL had not been able to identify all the information contained in the source and target questions, such as information regarding suitable trigonometric identities which could be applied to solve the target questions, correctly. This could be because PU and AL lacked a good understanding of the concept. Their memories of past material, namely the form of quadratic equations, were still weak, whereas in analogical reasoning, solving problems would involve relating past knowledge to the problems to be solved [16]. Thus, at the structuring and mapping stages, students with moderate mathematical creative thinking ability were not considered to be able to carry out the structuring and mapping excellently as there were students who did poorly in identifying information, such as trigonometric identities as well as the quantifiers and denominators in trigonometric equation fractions, in source and target problems.

On the other hand, students with low creative thinking ability, NH, TM, and ZH, were not able to perform structuring and mapping properly at all when identifying trigonometric identities to solve trigonometric equation problems. However, when it came to identifying the forms of quadratic equations and trigonometric equations, ZH and TM were quite good. They were also able to mention the relationship of information between the two. As for NH, she was unable to do so, and when asked for confirmation in an interview, she admitted that she copied her friends' answers. It is assumed that the errors in analogical reasoning were due to a lack of analysis when connecting the conditions of the source problems and those of the target problems [23]. In the mapping stage, the students made mistakes in identifying the relationship between the source and target questions of the same object [20].

In the applying and verifying stages, students were asked to solve the problems presented and draw conclusions. LF and PU were able to carry out the applying and verifying stages well by applying the information previously obtained from the source problems to the target problems. Meanwhile, AL could not perform verifying well, as can be seen from him only working on the example. He was unable to perform the reverse operation to arrive at the equation presented in the question. In addition, he did not confirm the angle value, which was the answer sought to the given problem, thus giving a different interpretation. In an interview, AL confirmed that he did not understand the concept of how to write the measure of an angle (e.g., x with $\sin(x)$). Students with low mathematical creative thinking ability, NH, TM, and ZH, on the other hand, could not perform the applying and verifying stages well. The students had a difficulty in not only finding answers based on the information in the source and target questions, but also in determining the relationship between the source and target questions. As revealed by research, students may make mistakes in several stages of analogical reasoning, namely, the stages of inferring and applying. In this study, students made mistakes in several stages of analogical reasoning too. In the applying stage of analogical reasoning, students made mistakes in adapting the steps to solve the source problems to the target problems.

As explained above, only LF could go through the four stages of analogical reasoning, namely structuring, mapping, applying, and verifying. PU could carry out the applying and verifying stages but failed to carry out the structuring and mapping stages, precisely in linking ordinary quadratic equations to trigonometric quadratic equations. Meanwhile, AL was not able to carry out all the analogical reasoning stages. AL made many mistakes in writing down mathematical symbols such as large angles and trigonometric forms. He also made errors in multiplying positive integers by negative integers when factoring. Thus, it can be concluded that students with moderate mathematical creative thinking abilities do not necessarily have high analogical reasoning abilities. This finding is in line with the statement of [1] that not all students in the high analogical reasoning ability category have high creative thinking ability, and not all students in the low analogical reasoning ability category have low creative thinking ability. However, when the ability to think creatively in mathematics is lacking, students will not be able to perform the analogical reasoning stages. Analogical reasoning errors are not a matter of reasoning itself. The question support could also lead students to make mistakes in solving mathematical problems. The form and level of difficulty of the questions could affect students' answers. Students prefer solving problems that are easy and familiar to solving new problems that will require past conceptual knowledge [15].

4 Conclusions

Analogical reasoning has a very important role in mathematics learning, precisely in solving problems that involve the application of past knowledge to the problem to be solved. In theory, analogical reasoning can develop students' critical and creative abilities. This means that analogy and creativity have a very close relationship. Research on analogical reasoning has not been widely carried out in mathematics education. Research on this topic has mostly been carried out in fields such as physics and biology. Students with low mathematical creative thinking ability are not necessarily unable to perform analogical reasoning well. Instead, through analogical reasoning, their mathematical creative thinking ability will grow. The stages of analogical reasoning help students think creatively by enabling them to find analogical forms that they can apply to solve target problems. Almost all problems in everyday life require creative thinking and analogical reasoning processes, but educators have not optimally incorporated them in students' problem-solving in learning. Therefore, it is necessary to increase the use of analogies in mathematics learnings. The limitation of this study lay in the use of questions that were not relevant to everyday life, whereas the form and level of difficulty of the questions could influence how students answered the questions. In addition, due to the COVID-19 pandemic, interviews were conducted online, and therefore, the character of each informant in answering was not revealed to a high degree.

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References

[1] D. R. Adiwijayanti, E. Yusmin & D. Astuti (2018). Kemampuan berpikir kreatif ditinjau dari kemampuan analogi dalam menyelesaikan masalah open-ended di SMP. *Jurnal Pendidikan dan Pembelajaran Khatulistiwa*, *7*(7), 1–8.

[2] E. M. Anderson, Y.-J. Chang, S. Hespos & D. Gentner (2018). Comparison within pairs promotes analogical abstraction in three-month-olds. *Cognition*, *176*, 74–86.

[3] A. Astuti, S. B. Waluya & M. Asikin (2020). Instrumen kemampuan berpikir kreatif matematika untuk siswa kelas IV sekolah dasar. *Musamus Journal of Primary Education*, *3*(1), 27–34. https://doi.org/10.35724/musjpe.v3i1.3117.

[4] L. Azaryahu, O. Broza, S. Cohen, S. Hershkovitz & E. Adi-Japha (2023). Development of creative thinking patterns via math and music. *Thinking Skills and Creativity*, *47*, 101196. urlhttps://doi.org/10.1016/j.tsc.2022.101196.

[5] U. Q. Azizah, E. Rooselyna & M. Masriyah (2021). Student's analogical reasoning in solving trigonometric problems in terms of cognitive style: A case study. *International Journal for Educational and Vocational Studies*, *3*(1). https://doi.org/10.29103/ijevs.v3i1.3398.

[6] R. A. Beghetto (2019). Creativity in the classroom. In J. C. Kaufman & R. J. Sternberg (Eds.), *The Cambridge handbook of creativity*, pp. 587–606. Cambridge University Press, Cambridge. https://doi.org/10.1017/9781316979839.029.

[7] A. B. Bernardo (2001). Analogical problem construction and transfer in mathematical problem solving. *Educational Psychology*, *21*(2), 137–150. https://doi.org/10.1080/01443410020043841.

[8] A. Bicer, Y. Lee, C. Perihan, M. M. Capraro & R. M. Capraro (2020). Considering mathematical creative self-efficacy with problem posing as a measure of mathematical creativity. *Educational Studies in Mathematics*, *105*, 457–485. https://doi.org/10.1007/s10649-020-09995-8.

[9] L. D. English (2004). *Mathematical and analogical reasoning of young learners*. Routledge, New York. https://doi.org/10.4324/9781410610706.

[10] M. S. Gary, R. E. Wood & T. Pillinger (2012). Enhancing mental models, analogical transfer, and performance in strategic decision making. *Strategic Management Journal*, *33*(11), 1229–1246. https://doi.org/10.1002/smj.1979.

[11] D. Gentner (2003). Why we're so smart. In D. Gentner & S. Goldin-Meadow (Eds.), *Language in mind: Advances in the study of language and thought*, pp. 195–235. MIT press, Cambridge.

[12] H. Gust, U. Krumnack, K.-U. Kühnberger & A. Schwering (2008). Analogical reasoning: A core of cognition. *Künstliche Intell.*, *22*(1), 8–12.

[13] S. J. Hespos, E. Anderson & D. Gentner (2020). Structure-mapping processes enable infants' learning across domains including language. In J. B. Childers (Ed.), *Language and Concept Acquisition from Infancy Through Childhood: Learning from Multiple Exemplars*, pp. 79–104. Springer International Publishing, Cham. https://doi.org/10.1007/978-3-030-35594-4_5.

[14] D. R. Hofstadter & E. Sander (2013). *Surfaces and essences: Analogy as the fuel and fire of thinking*. Basic Books, New York, NY.

[15] K. Kristayulita, T. Nusantara, A. R. Asari & C. Sadijah (2020). Schema of analogical reasoning-thinking process in example analogies problem. *Eurasian Journal of Educational Research*, *20*(88), 87–104.

[16] D. R. Kusumawardani, W. Wardono & K. Kartono (2018). Pentingnya penalaran matematika dalam meningkatkan kemampuan literasi matematika. In *Prisma, Prosiding Seminar Nasional Matematika*, volume 1 pp. 588–595. Universitas Negeri Semarang, Semarang. https://journal.unnes.ac.id/sju/index.php/prisma/article/view/20201.

[17] R. Leikin (2009). *Exploring Mathematical Creativity Using Multiple Solution Tasks*, pp. 129 – 145. Brill, Leiden, The Netherlands. https://doi.org/10.1163/9789087909352_010.

[18] I. Magdas (2015). Analogical reasoning in geometry education. *Acta Didactica Napocensia*, *8*(1), 57–65.

[19] N. Mandasari & E. Rosalina (2021). Analysis of students' difficulties in solving integer operations problems in elementary schools. *Research & Learning in Elementary Education*, *5*(3).

[20] I. G. B. Manuaba, A. Sutawidjaja & H. Susanto (2018). Kesalahan penalaran analogi siswa kelas XII sma dalam memecahkan masalah nilai maksimum. In *Seminar Nasional Pendidikan Matematika Ahmad Dahlan*, volume 1 pp. 105–115.

[21] L. Muhamad Safiih & N. A. M. A. (2016). Confirmatory factor analysis approach: A case study of mathematics students' achievement in TIMSS. *Malaysian Journal of Mathematical Sciences*, *10*(S), 41–51.

[22] C. C. Navarrete (2013). Creative thinking in digital game design and development: A case study. *Computers & Education*, *69*, 320–331.

[23] A. W.-K. Pang & J. Dindyal (2009). Analogical reasoning errors in mathematics at junior college level. In *Proceedings of the 32nd Annual Conference of the Mathematics Education Research Group of Australasia*, volume 1.

[24] E. Pehkonen (1997). The state-of-art in mathematical creativity. *Zentralblatt fur Didaktik der Mathematik*, *3*(29), 63–67.

[25] L. E. Richland & I. M. McDonough (2010). Learning by analogy: Discriminating between potential analogs. *Contemporary Educational Psychology*, *35*(1), 28–43. https://doi.org/10.1016/j.cedpsych.2009.09.001.

[26] M. Ruppert (2013). Ways of analogical reasoning–thought processes in an example based learning environment. In *Eighth Congress of European Research in Mathematics Education*, pp. 6–10.

[27] S. Said-Metwaly, W. V. den Noortgate & E. Kyndt (3917). Methodological issues in measuring creativity: A systematic literature review. *Creativity Theories - Research - Applications*, *4*(2), 276–301.

[28] G. Scott, L. E. Leritz & M. D. Mumford (2004). The effectiveness of creativity training: A quantitative review. *Creativity research journal*, *16*(4), 361–388. https://psycnet.apa.org/doi/10.1207/s15326934crj1604_1.

[29] A. Shriki (2013). A model for assessing the development of students' creativity in the context of problem posing. *Creative Education*, *4*(7), 430. http://dx.doi.org/10.4236/ce.2013.47062.

[30] N. K. Simms, R. R. Frausel & L. E. Richland (2018). Working memory predicts children's analogical reasoning. *Journal of Experimental Child Psychology*, *166*, 160–177. https://doi.org/10.1016/j.jecp.2017.08.005.

[31] Sukestiyarno (2020). *Metode Penelitian Pendidikan*. UNNES Press, Indonesia.

[32] N. Syafriafdi, A. Fauzan, I. M. Arnawa, S. Anwar & W. Widada (2019). The tools of mathematics learning based on realistic mathematics education approach in elementary school to improve math abilities. *Universal Journal of Educational Research*, *7*(7), 1532–1536.

[33] E. S. Utomo, D. Juniati & T. Y. E. Siswono (2018). Exploring aspects of mathematical visualization of junior high school student in a problem-solving task. *Malaysian Journal of Mathematical Sciences*, *12*(3), 421–436.

[34] L. S. Vygotsky (2004). Imagination and creativity in childhood. *Journal of Russian & East European Psychology*, *42*(1), 7–97.